

# Consensus Based Distributed Change Detection Algorithms

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**Abstract** – In this paper a consensus based distributed recursive algorithm based on geometric moving average control charts is proposed for change detection while monitoring environment by sensor networks. Convergence of the algorithm to the optimal centralized solution is proved in the cases of constant and time varying forgetting factors assuming correlated data and different parameters attached to the nodes in the network. Using the same methodology, a consensus based recursive Generalized Likelihood Ratio scheme is proposed for change detection. Experimental results illustrate characteristic properties of the algorithms.

**Keywords:** sensor networks, distributed detection, change detection, recursive algorithms, generalized likelihood ratio, convergence.

## 1 Introduction

A great deal of attention has been paid recently to signal processing with *distributed sensors*, having in mind the low cost of sensors, the availability of high speed *networks* and increased computational capabilities, *e.g.* [15, 16]. One of typical tasks of *sensor networks* is *distributed detection* [4, 16, 10]. In classical multi-sensor detection all the local sensors send all their data to other sensors, and ultimately to a *fusion center*. This topology may be too restrictive in many applications. Some advantages of the distributed signal processing may be increased reliability, reduced communication bandwidth requirements and reduced cost. However, this may result into a certain loss of performance with respect to the centralized system. In the case of detection of changes in the monitored environment, it is often desirable to have a possibility to test decision variables in real time at any node in the network, *e.g.* [2, 3, 4].

*Consensus techniques* have been in the focus of researchers for many years, starting from early 80's when important results were obtained in the areas of distributed asynchronous iterations in parallel computation

and distributed optimization, *e.g.* [14, 7]. There have been some recent attempts to apply consensus techniques to the distributed detection problem [6]. However, the underlying assumption is that the dynamic agreement process starts after all data have been collected, implying inapplicability to real time change detection problems. In [11, 12, 13] algorithms for distributed state and parameter estimation have been proposed by combining local overlapping decentralized estimation schemes with the dynamic consensus algorithm. Analogous algorithms for distributed detection based on "running consensus" have been proposed and discussed in [2, 3].

In this paper an algorithm is proposed for *distributed change detection* while monitoring the environment in wireless sensor networks. It is assumed that all the nodes in the network can generate local decision variables by recursive schemes belonging to *geometric moving average control charts* [1]. By applying a dynamic consensus scheme with time varying random communication gains one obtains an algorithm which asymptotically provides nearly equal behavior of all the nodes, *i.e.* any node can be selected for testing its decision variable w.r.t. a pre-specified threshold. It is proved that the mean square error between the optimal centralized decision variable and the local ones generated by the proposed algorithm behaves like  $O((1 - \alpha)^2)$ , where  $\alpha$  is the forgetting factor of the algorithm. The network gains are assumed to be time varying, allowing *e.g.* "gossip" algorithms [2]. In the case of time varying forgetting factors, it is proved that the algorithm converges to the optimal centralized scheme in the mean-square sense under very general conditions by using stochastic approximation arguments. The results analogous to those given in [2, 3] are derived as a special case. The same methodology has been applied to obtain a consensus based recursive Generalized Likelihood Ratio algorithm [1, 5]. Some experimental results are given as an illustration of the efficiency of the proposed algorithm.

## 2 Distributed Change Detection Algorithm

Consider a sensor network containing  $n$  nodes, where each node collects measurements and generates at each discrete time instant  $t$  a scalar quantity  $x_i(t)$ ,  $i = 1, \dots, n$ , which represents a realization of a random function of measurements generated by any preselected local data acquisition and signal processing unit;  $\{x_i(t)\}$  will be considered as mutually independent stationary random sequences with means  $E\{x_i(t)\} = m_i$  and covariances  $r_i(\tau) = E\{(x_i(t) - m_i)(x_i(t + \tau) - m_i)\}$ .

We shall assume that the network is aimed at change detection purposes and that the global decision function for the whole network is defined by

$$s_c(t+1) = \alpha s_c(t) + \frac{1-\alpha}{n} \sum_{i=1}^n x_i(t+1), \quad s_c(0) = 0 \quad (1)$$

where  $0 < \alpha < 1$ . Algorithm (1) is a representative of a class of on-line change detection algorithms belonging to geometric moving average control charts [1]. Obviously, the global detection procedure is based on testing the decision function  $s_c(t)$  with respect to an appropriately chosen threshold  $\lambda_c > 0$ , so that a change is detected when, *e.g.*,  $|s_c(t)| > \lambda_c$ . Notice that the algorithm requires a fusion center.

The aim of this paper is to propose a consensus based distributed change detection algorithm which does not require a fusion center and in which the output of any preselected node can be used as a representative of the whole network and tested w.r.t. a pre-specified common threshold. The basic assumption for this algorithm is that the nodes of the network are connected in such a way that the  $n \times n$  matrix  $C(t)$  represents the time varying weighted adjacency matrix for the underlying graph representing the network, and that  $C(t)$  is doubly stochastic for each  $t$ ; we shall assume, additionally, that matrices  $C(t)$  are random, iid and statistically independent from the sequences  $x_i(t)$ ,  $i = 1, \dots, n$ . We propose the following algorithm for generating the vector decision function  $s(t) = [s_1(t) \dots s_n(t)]^T$ :

$$s(t+1) = \alpha C(t)s(t) + (1-\alpha)C(t)x(t+1), \quad s(0) = 0 \quad (2)$$

where  $x(t) = [x_1(t) \dots x_n(t)]^T$ . The algorithm is derived from the consensus based state and parameter estimation algorithms proposed in [11, 12, 13]; it is also similar to the detection algorithm based on time averaging proposed in [2, 3]. Notice that the consensus matrix  $C(t)$  performs for each node "convexification" of the neighboring states and enforces in such a way consensus between the nodes. After achieving  $s_i(t) \approx s_j(t)$ ,  $i, j = 1, \dots, n$ , change detection can be done by testing  $|s_i(t)|$  for any  $i$  with respect to  $\lambda_c$ , provided (2) achieves a good approximation of  $s_c(t)$  generated by (1).

Define the error between the vector  $s(t)$  and the state

of the optimal centralized scheme as

$$e(t) = s(t) - \mathbf{1}s_c(t) = (I - \frac{\mathbf{1}\mathbf{1}^T}{n})s(t), \quad (3)$$

where  $\mathbf{1}^T = [1 \dots 1]$ .

Relation (3) can be demonstrated directly after noting that (2) gives

$$s(t) = (1-\alpha) \sum_{i=0}^{t-1} \alpha^i \phi(t-1, t-1-i)x(t-i) \quad (4)$$

after iterating back to the initial condition, where  $\phi(t, i) = C(t)C(t-1) \dots C(i)$  for  $t \geq i$  (compare with [2]); also,

$$s_c(t) = \frac{(1-\alpha)}{n} \sum_{i=0}^{t-1} \alpha^i \mathbf{1}^T x(t-i), \quad (5)$$

so that  $s_c(t) = \frac{\mathbf{1}^T s(t)}{n}$ , and (3) follows.

Starting from (3) we obtain, similarly as in [2], the following recursion for the error

$$e(t+1) = \alpha \tilde{C}(t)e(t) + (1-\alpha)\tilde{C}(t)\tilde{x}(t+1), \quad e(0) = 0 \quad (6)$$

where  $\tilde{C}(t) = C(t) - \frac{\mathbf{1}\mathbf{1}^T}{n}$  and  $\tilde{x}(t) = (I - \frac{\mathbf{1}\mathbf{1}^T}{n})x(t)$ . Consequently,

$$e(t) = (1-\alpha) \sum_{i=0}^{t-1} \alpha^i \tilde{\phi}(t-1, t-1-i)\tilde{x}(t-i), \quad (7)$$

where  $\tilde{\phi}(t, i) = \tilde{C}(t)\tilde{C}(t-1) \dots \tilde{C}(i)$  for  $t \geq i$  (notice that  $\tilde{C}(t)(I - \frac{\mathbf{1}\mathbf{1}^T}{n}) = \tilde{C}(t)$ ).

## 3 Convergence Analysis

We shall analyze properties of the proposed algorithm starting from the expression for the error process  $e(t)$  in (3).

We first realize that  $s(t)$  as the estimator of  $s_c(t)$  is, in general, biased, *i.e.*, we have

$$E\{e(t)\} = (1-\alpha) \sum_{i=0}^{t-1} \alpha^i E\{\tilde{\phi}(t-1, t-1-i)\}(m - \mathbf{1}\bar{m}), \quad (8)$$

where  $E\{\tilde{\phi}(t, i)\} = E\{\tilde{C}(t)\}E\{\tilde{C}(t-1)\} \dots E\{\tilde{C}(i)\}$ ,  $m = [m_1, \dots, m_n]^T$  and  $\bar{m} = \frac{1}{n} \sum_{j=1}^n m_j$ . Obviously,  $E\{e(t)\} = 0$  when  $m_i = m_j$ ,  $i, j = 1, \dots, n$ , having in mind that the expression in the sum cannot be equal to zero, in general (compare with [2], where it is adopted that  $m = 0$ ). The bias is, obviously, smaller when  $\alpha$  is closer to one.

The focus of the analysis is placed on the mean-square error matrix, defined, according to (6), as

$$Q(t) = E\{e(t)e(t)^T\} = (1-\alpha)^2 E\{\tilde{\Phi}(t)^T R \tilde{\Phi}(t)\}, \quad (9)$$

where

$$\tilde{\Phi}(t) = [\alpha^{t-1}\tilde{\phi}(t-1, 0): \alpha^{t-2}\tilde{\phi}(t-1, 1): \dots : \tilde{\phi}(t-1, t-1)],$$

while  $R = E\{\tilde{X}(t)\tilde{X}(t)^T\}$ , where  $\tilde{X}(t) = [\tilde{x}(1)^T: \dots : \tilde{x}(t)^T]^T$ . Moreover, we have that  $R = [R_{ij}]$ ,  $i, j = 1, \dots, t$ , where  $R_{ij}$  are  $n \times n$  block matrices defined as

$$R_{ij} = \text{diag}\{r_1(i-j), \dots, r_n(i-j)\} + mm^T = R_{ij}^{(1)} + R_{ij}^{(2)}. \quad (10)$$

Remark that (9) follows from (6) after noticing that  $\tilde{C}(t)\tilde{x}(t+1) = \tilde{C}(t)x(t+1)$ , since  $\tilde{C}(t)\mathbf{1}\mathbf{1}^T = 0$ .

*Theorem 1.* Let the following assumptions hold:

A1)  $E\{C(t)C(t)^T\}$  has the eigenvalue 1 with multiplicity 1;

A2)  $\max_i \sum_{\tau=0}^{\infty} |r_i(\tau)| < K < \infty$ .

Then, under the adopted assumptions,

$$\max_{i,j} Q_{ij}(t) = O((1-\alpha)^2),$$

where  $Q_{ij}(t)$  are the elements of  $Q(t)$  in (9).

*Proof:* Consider an arbitrary deterministic vector  $y$  orthogonal to  $\mathbf{1}$  and analyze the quadratic form

$$y^T Q(t)y = (1-\alpha)^2 y^T E\{\tilde{\Phi}(t)^T R \tilde{\Phi}(t)\}y \quad (11)$$

(otherwise  $y^T Q(t)y$  may be reduced to zero, see [2]). Obviously,

$$\lambda_{\max}(R_{ij}) \leq \lambda_{\max}(R_{ij}^{(1)}) + \lambda_{\max}(R_{ij}^{(2)}).$$

Furthermore,

$$\lambda_{\max}(R_{ij}^{(1)}) \leq \|R_{ij}^{(1)}\|_{\infty} < K < \infty$$

by assumption A.2 ( $\|A\|_{\infty} = \max_i \sum_j |a_{ij}|$ , where  $A = [a_{ij}]$  is a given matrix), and

$$\lambda_{\max}(R_{ij}^{(2)}) \leq m^T m.$$

The expression  $y^T E\{\tilde{\Phi}(t)^T \tilde{\Phi}(t)\}y$  is in the form of a sum of the terms containing  $y^T E\{\tilde{\phi}(t-1, i)^T \tilde{\phi}(t-1, i)\}y$ ,  $i = 0, \dots, t-1$ . By assumption A.1, matrix  $E\{\tilde{C}(t)\tilde{C}(t)^T\}$  is symmetric and positive semidefinite with the maximal eigenvalue  $\lambda_M$ ,  $0 \leq \lambda_M < 1$ . Therefore,

$$y^T E\{\tilde{\phi}(t-1, i)^T \tilde{\phi}(t-1, i)\}y \leq \lambda_M^i \|y\|^2,$$

so that

$$y^T E\{\tilde{\Phi}(t)^T \tilde{\Phi}(t)\}y \leq \|y\|^2 (K + m^T m) \sum_{i=0}^{\infty} \alpha^i \lambda_M^i < K_1 < \infty. \quad (12)$$

Consequently, choosing  $y = e_i - \mathbf{1}/n$ , where  $e_i$  denotes the vector of zeros with only the  $i$ -th entry equal to one (see [2]), one obtains

$$Q_{ii}(t) \leq K_1(1-\alpha)^2.$$

Using now  $y = e_i - e_j$ ,  $i \neq j$ , we show that  $(Q_{ii} + Q_{jj})/2 - Q_{ij} \leq K_1(1-\alpha)^2$ . Thus the result. Q.E.D.

The meaning of the obtained result becomes clearer after showing that for  $\alpha$  close to 1 the mean-square error between the states of the proposed distributed detection system is much smaller ( $O(1-\alpha)^2$ ) than the mean-square value of the state of the centralized system ( $O(1-\alpha)$ ). Namely, it follows from (5) that

$$E\{s_c(t)^2\} \leq (K + m^T m)(1-\alpha)^2 \sum_{i=0}^{\infty} \alpha^{2i} = O(1-\alpha). \quad (13)$$

*Example 1. - Convergence Analysis of the Distributed Change Detection Algorithm.* Let us consider a sensor network with  $n = 5$  nodes, where the means  $m_i$  and variances  $\sigma_i^2$  are randomly taken from the interval  $[0, 1]$  ( $m_i = 0$  in the case of no change). A protocol in which at each discrete time instant two randomly selected nodes exchange their data is selected. The moment of change is chosen to be  $t = 200$ . The values of  $Q_{ii}(t)$  are estimated for different values of  $\alpha$  for  $t = 1000$  using 1000 Monte Carlo runs. Figure 1 entirely confirms the above analysis.

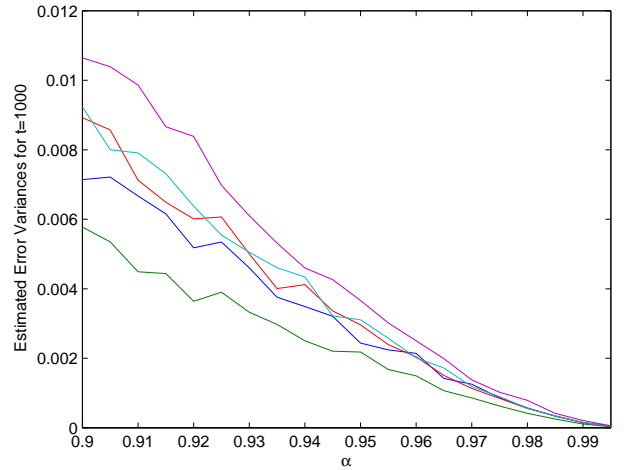


Figure 1: Estimated error variances as functions of  $\alpha$

## 4 Distributed Detection Based on Averaging

The recursive algorithms (1) and (2) with constant coefficient  $\alpha$  are essentially tracking algorithms with exponential forgetting, able to cope with change detection

phenomena [1]. The same form of the algorithms can be used also in the case when detection has to be based on time averaging on infinite intervals. In this case we shall assume that  $\alpha$  is a function of time tending to 1 when  $t$  tends to infinity, and we shall expect that the error between the states of the algorithms (1) and (2) converges to zero in the mean-square sense. However, the algorithms are then not directly suitable for change detection purposes. Notice that, in general, both algorithms (1) and (2) can be considered as stochastic approximation algorithms. Stochastic approximation algorithms with consensus, representing a generalization of (2) to the regression problem, have been analyzed in [13] starting from the basic results presented in [14].

*Theorem 2.* Let in (1) and (2)  $\alpha$  be replaced by  $\alpha(t+1) = 1 - \gamma(t+1)$ , and let the assumptions A1 and A2 be satisfied, together with:

A3)  $\gamma(t) > 0$ ,  $\sum_{t=1}^{\infty} \gamma(t) = \infty$ ;  $\sum_{t=1}^{\infty} \gamma(t)^2 < \infty$ .

Then,

$$\|Q(t)\| = o(1).$$

( $o(1)$  stands for a sequence tending to zero as  $t$  tends to infinity).

*Proof:* Starting from (2) and (4) one obtains (3). Consequently,

$$e(t) = \sum_{i=0}^{t-1} \pi(t, t-i) \tilde{\phi}(t-1, t-1-i) \gamma(t-i) \tilde{x}(t-i), \quad (14)$$

where  $\pi(t, i) = \alpha(t) \cdots \alpha(i+1)$  for  $t > i$  and  $\pi(t, t) = 1$ .

Following the above line of thought, we calculate  $Q(t) = E\{e(t)e(t)^T\}$  and obtain, similarly as in (11), the following expression

$$y^T Q(t) y = y^T E\{\tilde{\Psi}(t)^T R \tilde{\Psi}(t)\} y, \quad (15)$$

where

$$\tilde{\Psi}(t) = [\pi(t, 1) \tilde{\phi}(t-1, 0) \gamma(1) :$$

$$\pi(t, 2) \tilde{\phi}(t-1, 1) \gamma(2) : \cdots : \pi(t, t) \tilde{\phi}(t-1, t-1) \gamma(t)].$$

Reasoning like in the proof of Theorem 1, we obtain

$$y^T Q(t) y \leq \|y\|^2 (K + m^T m) \sum_{i=0}^t \pi(t, t-i)^2 \lambda_M^i \gamma(t-i)^2. \quad (16)$$

Now it is possible to apply the Kronecker's lemma (see [13]) and to conclude immediately that

$$\lim_{t \rightarrow \infty} \sum_{i=0}^t \pi(t, t-i)^2 \lambda_M^i \gamma(t-i)^2 = 0,$$

wherefrom the result. Q.E.D.

The result of Theorem 2 can be applied, obviously, to the special case when  $\gamma(t) = \frac{1}{t}$ , treated in [2, 3] under the assumption that the sequences  $\{x_i(t)\}$ ,  $i = 1, \dots, n$ , are mutually independent and iid. In this case, the

main difference between the algorithm (2) with  $\alpha(t+1) = 1 - \frac{1}{t+1}$  and the algorithm analyzed in [2] lies in the fact that we apply the "convexification" operator not only to the previous detector state, but also to the measurement term, leading to an additional smoothing effect.

*Corollary 1.* Under the assumptions of Theorem 2 and with  $\gamma(t) = \frac{1}{t}$  we have

$$\|Q(t)\| = O(t^{-2}), \quad (17)$$

while  $E\{s_c(t)^2\} = O(t^{-1})$ .

*Proof:* In this special case, we have

$$e(t) = \frac{1}{t} \sum_{i=0}^{t-1} \tilde{\phi}(t-1, t-1-i) \tilde{x}(t-i), \quad (18)$$

and the result immediately follows after applying the methodology of Theorems 1 and 2. Notice that  $E\{s_c(t)^2\}$  can be easily calculated using

$$s_c(t) = \frac{1}{nt} \sum_{i=0}^{t-1} \mathbf{1}^T x(t-i). \quad (19)$$

Q.E.D.

## 5 Recursive Distributed GLR Algorithm

In this Section we shall describe and analyze a recursive distributed change detection algorithm based on consensus derived from the Generalized Likelihood Ratio (GLR) methodology applied to the detection of changes of the mean of a stationary random process, e.g. [1]. Assume that we have the following system model:

$$y_i(t) = m_i + \epsilon_i(t), \quad (20)$$

where  $\epsilon_i(t) \sim N(0, \sigma_i^2)$ ,  $i = 1, \dots, n$ ,  $\{\epsilon_i(t)\}$  are mutually independent iid processes, while  $m_i = 0$  in the case of no change, and  $m_i = \theta_i \neq 0$  in the case of change. In the case when  $\theta_i$  is not a priori known, the application of the GLR methodology leads to the following statistics based on  $N$  successive measurements ( $t = 1, \dots, N$ ) [5]

$$s_i(N) = \frac{N}{2} \bar{y}_i(N)^2 \sigma_i^{-2}, \quad (21)$$

where  $\bar{y}_i(N) = \frac{1}{N} \sum_{t=1}^N y_i(t)$ .

Introducing  $t$  for current time, we obtain, using [5], the following basic local recursions attached to the nodes of the network

$$s_i(t+1) = \frac{t}{t+1} s_i(t) + \frac{\sigma_i^{-2}}{t+1} [(t+1) \bar{y}_i(t+1) - \frac{1}{2} y_i(t+1)] y_i(t+1). \quad (22)$$

Denoting  $x_i(t) = \sigma_i^{-2} [t \bar{y}_i(t) - \frac{1}{2} y_i(t)] y_i(t)$  and  $x(t) = [x_1(t) \cdots x_n(t)]^T$  we can come back to the recursion (1)

to define an on-line centralized GLR change detection algorithm, and to the recursion (2) to define an on-line change detection GLR algorithm based on consensus (after replacing  $\frac{t}{t+1}$  by  $\alpha$  and  $\frac{1}{t+1}$  by  $1 - \alpha$ ). Both obtained algorithms are structurally identical to those discussed above; however, their properties differ significantly from the properties of (1) and (2) as a consequence of the specific definition of  $x_i(t)$ . Indeed, taking  $\alpha(t+1) = 1 - \frac{1}{t+1}$  and coming back to (22), one obtains that the recursion for the error (6) between the centralized scheme and the distributed one becomes now

$$e(t+1) = \frac{t}{t+1} \tilde{C}(t)e(t) + \frac{1}{t+1} \tilde{C}(t) \Sigma^{-1} [(t+1) \bar{Y}(t+1) - \frac{1}{2} Y(t+1)] y(t+1), \quad (23)$$

where  $\Sigma = \text{diag}\{\sigma_1^2, \dots, \sigma_n^2\}$ ,  $\bar{Y}(t) = \text{diag}\{\bar{y}_1(t), \dots, \bar{y}_n(t)\}$ ,  $Y(t) = \text{diag}\{y_1(t), \dots, y_n(t)\}$  and  $y(t) = [y_1(t) \dots y_n(t)]^T$ , while  $\bar{y}_i(t)$  is generated recursively by

$$\bar{y}_i(t+1) = \frac{t}{t+1} \bar{y}_i(t) + \frac{1}{t+1} y_i(t+1). \quad (24)$$

Obviously, in the case of tracking we have to replace simply  $\frac{1}{t+1}$  by  $1 - \alpha$  in both (23) and (24).

Properties of the proposed consensus based GLR change detection algorithm can be clarified using the methodology of Theorems 1 and 2, together with Corollary 1. The main point is now that the mathematical expectation of  $x(t)$  is not constant any more, but increases linearly with  $t$ , having in mind that  $\bar{Y}(t+1)$  in (23) is multiplied by  $t+1$ . Indeed, we can start from (9) and realize that the covariance matrix  $R$  is not constant any more. We can simply come to the conclusion about qualitative asymptotic behavior of the mean square error after retaining, for  $t$  large enough, the more significant term  $(t+1)\bar{Y}(t+1)$  in the bracket at the right hand side, and assuming that, approximately,  $\bar{Y}(t) \approx \theta + \nu(t)$ , where  $\theta = [\theta_1 \dots \theta_n]^T$  and  $\nu(t)$  is a white noise term. In this case it is easy to see that  $\lambda_{\max}(R(t)) \sim t^2$ . After coming back to Theorem 2 and Corollary 1, we directly conclude that the mean square error is asymptotically bounded. The importance of this conclusion becomes clear after realizing that the optimal centralized detector provides  $E\{s_c(t)^2\}$  which diverges linearly with  $t$ , *i.e.* the ratio between the mean square values of the error and the optimal statistic tends to zero as  $t$  tends to infinity.

In the case of change detection with constant  $\alpha$  we always have bounded decision functions; however, in this case the mean square error cannot be bounded by  $O((1 - \alpha)^2)$ , as in Theorem 1. In spite of that, the consensus based detector works very efficiently due to the fact that the application of the consensus based "convexification" achieves similar behavior of all the nodes in the network, together with better performance with respect to the majority of local detectors.

*Example 2. - Properties of the Recursive GLR Algorithm Based on Consensus.* A sensor network with  $n = 10$  nodes is considered and the means  $m_i$  and variances  $\sigma_i^2$  are randomly taken from the interval  $[0.5, 1.5]$  ( $m_i = 0$  in the case of no change). The moment of change is  $t = 200$ . A protocol in which at each discrete time instant two randomly selected nodes exchange their data is compared to the protocol when only the admissible pairs of nodes can be selected for data exchange. The nodes can be connected to randomly spatially distributed agents within a square area; two nodes are connected if their distance is less than a pre-determined threshold, (one realization of such spatial distribution is depicted in Fig. 2). The proposed algorithm effectively achieves very similar behavior of all of the nodes in both cases, with local decision functions getting closer to the global decision function as  $\alpha \rightarrow 1$ . Naturally, the protocol for the fully connected network shows better performance, as can be seen on Fig. 3 (left), where the global decision function is given by dashed lines (mean  $\pm$  one standard deviation), together with the decision function of one randomly selected node (solid lines), compared to the restricted topology case (right).

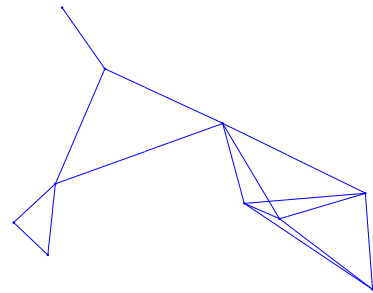
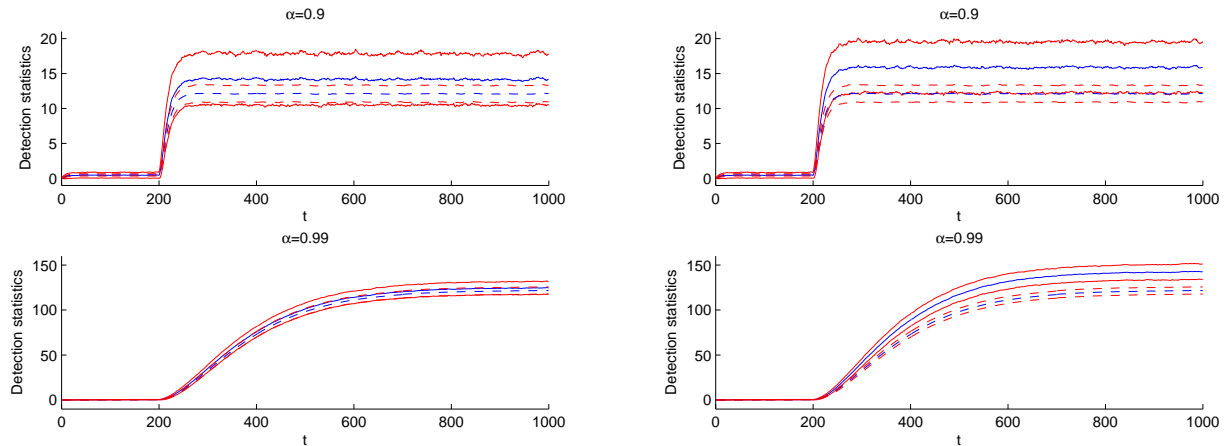


Figure 2: One realization of the network graph

## 6 Conclusion

In this paper a consensus based distributed recursive algorithm based on geometric moving average control charts is proposed for change detection while monitoring environment by sensor networks. Convergence of the algorithm to the optimal centralized solution is studied assuming correlated data and different local environments. A recursive consensus based distributed GLR algorithm is also constructed and discussed.

Further work can be oriented towards direct generalizations in the sense of allowing arbitrary weights in the global decision function, leading to networks characterized by nonsymmetric weighted adjacency matrices.



(a) Case of fully connected network topology

(b) Case when only admissible pairs of nodes exchange their data

Figure 3: Decision functions: for one node (solid lines) and global (dashed lines)

## References

- [1] M. Basseville and L. V. Nikiforov, *Detection of Abrupt Changes: Theory and Applications*, Prentice Hall, 1993.
- [2] P. Braca, S. Marano and V. Matta, “Enforcing Consensus While Monitoring the Environment in Wireless Sensor Networks”, *IEEE Trans. Signal Processing*, vol. 56, 2008, pp. 3375-3380.
- [3] P. Braca and S. Marano and V. Matta and P. Willett, “Asymptotic Optimality of Running Consensus in Testing Binary Hypotheses”, *IEEE Trans. Signal Processing*, 2010, Vol. 58, pp. 814-825.
- [4] J. F. Chamberland and V. Veeravalli, “Decentralized Detection in Sensor Networks”, *IEEE Trans. Signal Proc.*, 2003, Vol. 50, pp. 407-416.
- [5] S. X. Ding, *Model Based Fault Diagnosis Techniques - Design Schemes, Algorithms and Tools*, Springer Verlag, 2008.
- [6] E. Franco, R. Olfati-Saber, T. Parisini and M. M. Polycarpou, “Distributed Fault Diagnosis using Sensor Networks and Consensus Based Filters”, *Proc. 45th IEEE CDC Conf.*, 2006.
- [7] R. Olfati-Saber, A. Fax and R. Murray, “Consensus and Cooperation in Networked Multi-Agent Systems”, *Proceedings of the IEEE*, 2007, Vol. 95, pp. 215-233.
- [8] R. Olfati-Saber, E. Franco, E. Frazzoli, J. S. Sharma, “Belief Consensus and Distributed Hypothesis Testing in Sensor Networks”, *Workshop on Emb. Sensing and Control*, Notre Dame Univ., 2005.
- [9] R. Olfati-Saber and R. Murray, “Consensus problems in networks of agents with switching topology and time-delays”, *IEEE Trans. Autom. Control*, vol. 49, 2004, pp. 1520-1533.
- [10] A. Speranzon, C. Fischione and K. H. Johansson, “Distributed and collaborative estimation over wireless sensor networks”, *Proc. IEEE CDC Conf.*, 2006.
- [11] S. S. Stanković, M. S. Stanković, D. M. Stipanović, “Consensus based overlapping decentralized estimator”, *IEEE Trans. Autom. Control*, vol. 54, 2009, pp. 410-415.
- [12] S. S. Stanković, M. S. Stanković, D. M. Stipanović, “Consensus Based Overlapping Decentralized Estimation With Missing Observations and Communication Faults”, *Automatica*, vol. 45, 2009, pp. 1397-1406.
- [13] S. S. Stanković, M. S. Stanković and D. M. Stipanović, “Decentralized parameter estimation by consensus based stochastic approximation”, *Proc. 46th IEEE Conference on Decision and Control*, 2007, pp. 1535-1540.
- [14] J. N. Tsitsiklis, D. P. Bertsekas and M. Athans, “Distributed asynchronous deterministic and stochastic gradient optimization algorithms”, *IEEE Trans. Autom. Control*, vol. 31, 1986, pp. 803-812.
- [15] P. K. Varshney, *Distributed Detection and Data Fusion*, Springer, 1996.
- [16] R. Vishwanathan and P. Varshney, “Distributed Detection with Multiple Sensors: Part I - Fundamentals”, *Proc. of the IEEE*, 1997, Vol. 85, pp. 54-63.